to the shear-induced changes of the effective mass components in the field direction. It is interesting to note that these changes are of appreciable magnitude. We find $(1/m^*)(dm^*/dX)\approx 5\times 10^{-12}$ cm^2/dyn. Since m^* is the reduced mass, we cannot separate the contributions of the light hole mass and of the (111) electron mass. The fact that the shear dependence of the current beyond the Kane kink is quite small allows us to conclude that either the effective mass deformation occurs mostly on the (111) ellipsoids or that there is an accidental cancellation of the deformations of the light holes and the (000) conduction band extremum.

The bias dependence of the hydrostatic pressure coefficient shows a clearly resolved structure at small bias voltages which is almost symmetrical with respect to the forward and reverse bias direction. In the small bias range of direct tunneling the pressure coefficient $\Pi_p=\Delta I/3Ip$ is about $\Pi_p=-7.5\times 10^{-11}$ cm^2/dyn. Π_p changes to $\Pi_p=-4\times 10^{-11}$ cm^2/dyn at $V=\pm 6$ mV before it reaches a constant value in the TA phonon region. Introducing for brevity the notation $\Pi_p(TA,\pm V)$ and $\Pi_p(LA,\pm V)$ to represent the pressure coefficient in the TA and LA phonon regions, at positive and at negative biases, respectively, we can list the experimental values in these regions as follows: $\Pi_p(TA,+V)=-5.3$; $\Pi_p(TA,-V)=-5.7$; $\Pi_p(LA,+V)=-6.5$; and $\Pi_p(LA,-V)\approx -7.4$ in units of 10^{-11} cm^2/dyn.

It is to be expected that Π_p of the direct tunneling current at very small biases is different from that of the indirect tunneling current. Furthermore, one expects the following relationships to hold between the magnitudes of Π_p: (i) $\Pi_p(LA,+V)<\Pi_p(TA,+V)$ and $\Pi_p(LA,-V)>\Pi_p(TA,-V)$, and (ii) $\Pi_p(LA,+V)<\Pi_p(LA,-V)$ and $\Pi_p(TA,+V)<\Pi_p(TA,-V)$. The reason for this is the change in β of Eq. (2) produced (i) by the difference in energy of the TA and LA phonons and (ii) by the reversal of the sign of $\hbar\omega$ as the bias is reversed.

The observed stress coefficients obey relation (ii). The differences in Π_p as observed upon reversal of bias agree with Eq. (2) to within the experimental accuracy. However, the relation (i) is not satisfied. The average $\frac{1}{2}[\Pi_p(LA,+V)+\Pi_p(LA,-V)]$, instead of being equal to the equivalent average of the Π_p in the TA phonon regions as predicted by Eq. (2), is about 25% larger in absolute magnitude. If this difference is interpreted as resulting from a difference in β in the two-phonon regions, then the tunneling probability would be a factor of 100 larger in the TA than in the LA phonon region. It is considered unreasonable that a difference in the electron-phonon coupling constants of the two phonons could compensate for such a large factor to give the same magnitude of the current densities in the two-phonon regions as is observed experimentally. The change of Π_p at the transition from the TA to the LA phonon region is, therefore, not understood at present.

The dip in Π_p observed at ± 6 mV is probably due to the change in the phonon energy with pressure. This dip is reproduced fairly well by a calculation based on the value of $d\ln(\hbar\omega)/dp=-12\times 10^{-12}$ cm^2/dyn for the TA phonon. The details of this calculation and the comparison with experiment will appear elsewhere.

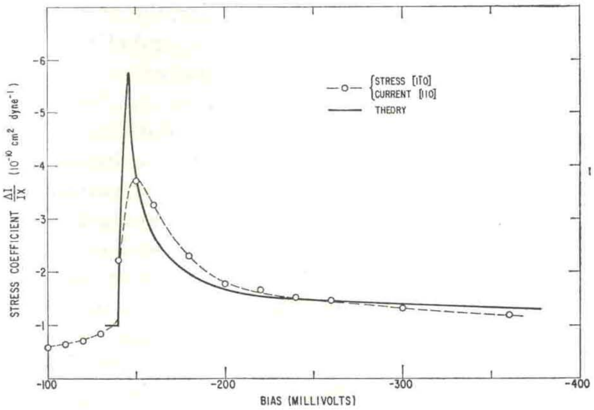

FIG. 6. Comparison between theoretical and experimental stress coefficient for reverse bias voltages beyond the Kane kink.

The pressure coefficient of sample 1 in the indirect tunneling range is due to the change of the indirect band gap $E_g(111)$ and that of the reduced mass m^*. Neglecting the minor stress variations of C_i and D one can write from Eq. (2)

$$\frac{\Delta I_i}{I_i}=-\beta\left[\frac{3}{2}\frac{\Delta E_g(111)}{E_g(111)}+\frac{1}{2}\frac{\Delta m^*}{m^*}-\frac{\Delta F}{F}\right]. \tag{13}$$

One may estimate β from Eq. (13) and the measured $\Pi_p=6\times 10^{-11}$ cm^2/dyn of sample 1 in the forward bias range. Here again it is difficult to estimate $\Delta m^*/m^*$. The reduced mass m^* in Eq. (13) contains the light hole mass and the component of the (111) valley mass tensor in the field direction. The change of the light hole mass can be estimated from Eq. (12) where $[1/E_g(000)]\times[dE_g(000)/dp]=12.4\times 10^{-12}$ cm^2/dyn. The change of the transverse electron mass, which predominantly determines the component of the electron mass in the field direction, is related to the band gap change at the zone face at [111] which is $(1/E_g)(dE_g/dp)=3.4\times 10^{-12}$ cm^2/dyn.[18] Hence, $(1/m^*)(dm^*/dp)=(8\pm 5)\times 10^{-12}$ cm^2/dyn may be a reasonable estimate. Using this value and the measured pressure coefficient one obtains $\beta=16\pm 3$ for $V=+60$ mV. The same calculation yields the value $\beta=20\pm 4$ for $V=-70$ mV. This increase of β with decreasing bias voltage disagrees strongly with Eqs. (2) and (3) which predict instead a slight decrease of β.[19]

One may think that this discrepancy is due to some simplifying assumptions on which the derivation of

[18] R. Zallen, W. Paul, and J. Tauc, Bull. Am. Phys. Soc. **7**, 185 (1962).

[19] This effect was also observed by M. I. Nathan and W. Paul, in *Proceedings of the International Conference Semiconductor Physics, Prague, 1960* (Czechoslovakian Academy of Sciences, Prague, 1961), p. 209.

Eqs. (2) and (3) is based, in particular, the assumptions of (i) a constant junction field, (ii) a parabolic shape of the energy bands in **k** space, and (iii) an abrupt transition from *n*-type to *p*-type doping. Nathan,[20] however, has calculated the tunneling exponent β using the more realistic field of a graded junction and including nonparabolic effects. He finds that both, the variation of the field with position in the junction and a graded impurity distribution, cause β to decrease with decreasing bias voltage even faster than predicted by Eqs. (2) and (3) and that the nonparabolic effects do not influence the bias dependence of β appreciably. In addition to this, Nathan finds that for interpreting the current-voltage characteristic of germanium diodes at 297°K one requires β to decrease with decreasing voltage. In comparing the current-voltage characteristic of the direct tunneling process beyond the Kane kink with Eqs. (2) and (3) in Fig. 5, we also concluded that α decreases with decreasing bias.

In view of this evidence it is difficult to understand the increase of β with decreasing bias voltage as observed in the hydrostatic pressure experiments.

Since one is dealing with the same diode, one should be able to calculate β from the previously determined $\alpha=17.6$ at $V=-300$ mV. Using Eqs. (1), (2), and Kane's value $\lambda_i/\lambda_d=16/3\pi$, one obtains for our case $\beta/\alpha=1.5$, and hence, $\beta=26.5$ at $V=-300$ mV. This value of β compares well with $\beta=23.5\pm5$ obtained by extrapolating the *observed* bias dependence of β to $V=-300$ mV. However, in view of the unexpected bias dependence of β, this agreement may be fortuitous.

The contribution to the stress coefficient of sample 2 which arises from the nonequivalence of the two pairs of valleys consists of two parts. (1) The shear causes some electrons to transfer from the two valleys which are raised in energy into the two which are lowered. Those which are lowered are the easy valleys, i.e., they have the smaller m^* in the tunneling direction. Hence we expect this part to give a positive contribution to Π. (2) The shift of the valleys will change also the tunneling probability because the energy gap for a particular valley is changed. Since the easy valleys are lowered in energy, their energy gap decreases under shear. This again results in a positive contribution to Π.

Since the first effect involves the density of states factor D, one has to write the current as a sum over the j valley contributions and the k-phonon contributions

$$I=\sum_{jk}I_{jk}, \tag{14}$$

where

$$I_{jk}=A_kD(eV\pm\hbar\omega_k/e)\exp(-\beta_j), \tag{15}$$

with

$$\beta_j=\lambda_i E_g^{3/2}(111)m_j^{*1/2}/F. \tag{16}$$

The plus and minus signs in Eq. (15) indicate that the phonon voltage $\hbar\omega_k/e$ has to be subtracted from the magnitudes of both the forward and the reverse bias voltage for calculating D_k. The coefficients A_k take into account the different electron-phonon interaction strengths for the two phonons.[12] From Fig. 3 we determined $A_2/A_1=2.23$, where A_2 refers to the higher energy phonon.

The stress coefficient due to the nonequivalence of the valleys will, then, be

$$\Delta I/IX=\sum_{jk}\Delta I_{jk}/\sum_{jk}I_{jk}X. \tag{17}$$

From Eq. (15) one obtains

$$\Delta I_{jk}=A_k\exp(-\beta_j)\times\left[\frac{dD_k}{d(\zeta_n)}(\Delta\zeta_n)_j-\frac{3}{2}\frac{D_k\beta_j}{E_g(111)}\Delta E_{gj}(111)\right]. \tag{18}$$

Equation (18) does not include the effect of shear on the transverse and longitudinal electron masses in the (111) valleys. This effect depends not only on the shifts of the conduction band valleys with shear but also on the equivalent quantities for the valence bands at the (111) zone face. These latter quantities are not accurately known at present. In any case, this effect would give rise to a contribution which is independent of bias similar to the second term in Eq. (18). It would, therefore, cause a change in the constant value which the theoretical curve approaches at large reverse bias, but it would not otherwise affect the bias dependence of the shear stress coefficient. Our conclusions would not be affected by the inclusion of this term.

Since we consider in (17) the pure shear part of the stress only we have

$$\sum_j(\Delta\zeta_n)_j=0,\quad \sum_j\Delta E_{gj}(111)=0, \tag{19}$$

and hence

$$(\Delta\zeta_n)_j=-\Delta E_{gj}(111). \tag{20}$$

For uniaxial compression along [1$\bar{1}$0] one finds

$$\Delta E_{g3}(111)=-\Delta E_{g1}(111)=\tfrac{1}{6}E_2S_{44}X. \tag{21}$$

Here the subscript 1 refers to the easy valleys $j=1,2$, and subscript 3 to the hard valleys $j=3,4$. $E_2=19$ eV is the deformation potential for pure shear[21] and $S_{44}=1.47\times10^{-12}$ cm^2/dyn is the elastic compliance constant.[22]

In the second term of Eq. (18), which is due to the change in tunneling probability, we have neglected the shear-induced change of the valley masses. Since the reduced mass is determined more by the light hole mass than by the electron mass, this effect is not large. Any change in the light hole mass, however, will affect both samples in the same way and hence will not contribute to the difference of Π of the two samples.

Because of the uncertainties involved in the previous determinations of α and β, we have adjusted the value of β_1 so that the magnitude of Eq. (17) agrees with the dif-

[20] M. I. Nathan, J. Appl. Phys. **33**, 1460 (1962).

[21] H. Fritzsche, Phys. Rev. **115**, 336 (1959).

[22] M. E. Fine, J. Appl. Phys. **24**, 388 (1953).